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PROBLEMS IN NONLINEAR ACOUSTICS:

SCATTERING OF SOUND BY SOUND, PARAMETRIC RECEIVING ARRAYS, NONLINEAR EFFECTS IN ASYMMETRIC SOUND BEAMS, AND PULSED FINITE AMPLITUDE SOUND BEAMS

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1 August 1989

First Annual Summary Report
ONR Grant N00014-89-J-1003

Prepared for:

OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY
ARLINGTON, VA 22217-5000

DISTRIBUTION STATEMENT A

89 8 17 110

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No 0704-0188

1a REPORT SECURITY CLASSIFICATION: Unclassified	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S): ASR-1	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION: University of Texas; 6b OFFICE SYMBOL (If applicable)	7a NAME OF MONITORING ORGANIZATION: Office of Naval Research
6c ADDRESS (City, State, and ZIP Code): Department of Mechanical Engineering, The University of Texas at Austin, Austin, TX 78712-1063	7b ADDRESS (City, State, and ZIP Code): Physics Division, Code 1112, Arlington, VA 22217-5000
8a NAME OF FUNDING/SPONSORING ORGANIZATION; 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: N00014-89-J-1003
8c ADDRESS (City, State, and ZIP Code)	10 SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO
61153N	4126943		

11 TITLE (Include Security Classification)
Problems in Nonlinear Acoustics: Scattering of Sound by Sound, Parametric Receiving Arrays, Nonlinear Effects in Asymmetric Sound Beams, and Pulsed Finite Amplitude Sound Beams

12 PERSONAL AUTHOR(S)
Mark F. Hamilton

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Annual Summary	FROM 880701 TO 890731	890801	18

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	
20	01		nonlinear acoustics; pulses; parametric array; scattering of sound by sound

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

Four projects are discussed in this annual summary report, all of which involve basic research in nonlinear acoustics. (1) Scattering of Sound by Sound, a theoretical study of two noncollinear Gaussian beams which interact to produce sum and difference frequency sound. (2) Parametric Receiving Arrays, a theoretical study of parametric reception in a reverberant environment. (3) Nonlinear Effects in Asymmetric Sound Beams, a numerical study of two dimensional finite amplitude sound fields. (4) Pulsed Finite Amplitude Sound Beams, a numerical time domain solution of the KZK equation.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT: [X] UNCLASSIFIED/UNLIMITED [ ] SAME AS RPT [ ] DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION: Unclassified
22a NAME OF RESPONSIBLE INDIVIDUAL: L. E. Hargrove, ONR Physics Division	22b TELEPHONE (Include Area Code): (202) 696-4221; 22c OFFICE SYMBOL: ONR Code 1112

DD Form 1473, JUN 86 — Previous editions are obsolete — SECURITY CLASSIFICATION OF THIS PAGE

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DTIC QUALITY INSPECTED 5

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INTRODUCTION

This annual summary report describes research performed from 1 July 1988 through 31 July 1989 with support from ONR. During the last ten months of this period (1 October 1988 through 31 July 1989), the support came from ONR grant N00014-89-J-1003. During the first three months (1 July 1988 through 30 September 1988), the support was provided by ONR contract N00014-85-K-0708.

The following projects are discussed in this report:

I. Scattering of Sound by Sound

II. Parametric Receiving Arrays

III. Nonlinear Effects in Asymmetric Sound Beams

IV. Pulsed Finite Amplitude Sound Beams

Contributions to these projects were made by the following individuals:

Senior Personnel

- M. F. Hamilton, principal investigator (projects I–IV)
- J. Naze Tjøtta, visiting scientist (projects I and II)
- S. Tjøtta, visiting scientist (projects I and II)

Graduate Students

- C. M. Darvennes, Ph.D. student in Mechanical Engineering (projects I and II)
- E. E. Kim, M.S. student in Mechanical Engineering (project III)
- Y.-S. Lee, Ph.D. student in Mechanical Engineering (project IV)

Naze Tjøtta and Tjøtta, who spent the past year at Applied Research Laboratories at the University of Texas at Austin (ARL:UT) while on leave from the University of Bergen, Norway, received no support from the ONR contract and grant covered by this report. Their support on the above projects was provided by ARL:UT, and VISTA/STATOIL of Norway. Kim received partial support from the National Science Foundation (2 months), and Lee received partial support from the Texas Advanced Research Program (8 months).

The following manuscripts and abstracts, which describe work supported at least in part by ONR, have been published (or submitted for publication) since 1 July 1988.

Refereed Publications

- M. F. Hamilton and J. A. TenCate, "Finite amplitude sound near cutoff in higher order modes of a rectangular duct," *J. Acoust. Soc. Am.* **84**, pp. 327–334 (1988).

- T. S. Hart and M. F. Hamilton, "Nonlinear effects in focused sound beams," *J. Acoust. Soc. Am.* 84, pp. 1488–1496 (1988).
- C. M. Darvennes and M. F. Hamilton, "Scattering of sound by sound from two Gaussian beams," submitted in March 1989 for publication in *J. Acoust. Soc. Am.*
- S. J. Lind and M. F. Hamilton, "Noncollinear interaction of a tone with noise," submitted in April 1989 for publication in *J. Acoust. Soc. Am.*

Publications in Conference Proceedings

- C. M. Darvennes, M. F. Hamilton, J. Naze Tjøtta, and S. Tjøtta, "Scattering of sound by sound: Effects of absorption," to appear in *Proceedings of the 13th International Congress on Acoustics*, Belgrade, Yugoslavia, August 1989.

Oral Presentation Abstracts

- Y.-S. Lee and M. F. Hamilton, "A parametric array for use as an ultrasonic proximity sensor in air," *J. Acoust. Soc. Am.* **84**, S8 (1988).
- C. M. Darvennes, M. F. Hamilton, J. Naze Tjøtta, and S. Tjøtta, "Effects of absorption on the scattering of sound by sound," *J. Acoust. Soc. Am.* **85**, S5 (1989).

I. SCATTERING OF SOUND BY SOUND

The effect of absorption on the scattering of sound by sound has been investigated theoretically by Darvennes, Naze Tjøtta, and Tjøtta. The contributions from Darvennes are closely related to her work on project II. Support for Naze Tjøtta and Tjøtta was provided by ARL:UT and VISTA/STATOIL.

A. Background

The scattering of sound by sound usually refers to the radiation of sum or difference frequency sound from a region where two harmonic sound beams interact at a nonzero angle. Background on investigations into the scattering of sound by sound over the past 30 years may be found in the second[1] and third[2] annual summary reports under ONR contract N00014-85-K-0708.

B. Results

The results obtained during the past year are summarized by the following abstract[3] of an oral presentation given at the 117th Meeting of the Acoustical Society of America in Syracuse, New York, on 23 May 1989.

The scattering of sound by sound in a lossless fluid was discussed at an earlier meeting.[4,5] Here, the effects of absorption are included. The Khokhlov-Zabolotskaya-Kuznetsov equation is used to derive farfield asymptotic results for the sum and difference frequency sound due to the noncollinear interaction of real sound beams radiated from displaced sources. There are two main contributions to the nonlinearly generated sound in the farfield: the continuously pumped sound and the scattered sound. Weak absorption affects neither the locations nor the relative amplitudes of the pumped and scattered difference frequency sound. Strong absorption attenuates the pumped difference frequency sound faster than the scattered difference frequency sound. The scattered sum frequency sound is always attenuated faster than the pumped sum frequency sound, and there may be shifts in the locations of the maxima. Numerical results are presented for the case of Gaussian primary beams.

The numerical results that follow are excerpted from a paper[6] to be published in the Proceedings of the 13th International Congress on Acoustics to be held in Belgrade, Yugoslavia during August 1989. We consider two Gaussian beams that intersect at an angle of 10°. The sources have the same radius a, their centers are displaced by $2a$, and the frequencies of the two beams differ by a factor of five. The thermoviscous absorption of the fluid is characterized in terms of the absorption length $L_\pm = (\alpha_1 + \alpha_2 - \alpha_\pm)^{-1}$, where α is the attenuation coefficient of the primary, sum, or difference frequency wave. The absorption length is positive for difference frequency generation and negative for sum frequency generation. Range is measured in terms of the dimensionless coordinate $Z = z/z_2$, where z_2 is the Rayleigh distance for the low frequency source. The dimensionless size of the low frequency source is given by $k_2 a = 30$, where k_2 is the associated wavenumber.

In Fig. 1 we show the effects of moderate absorption on sum and difference frequency beam patterns at ranges from $Z = 100$ to $Z = 500$. We have set $|L_\pm| = 100 z_2$, and therefore the dimensionless range $Z = 100$ corresponds to one absorption length. In the difference frequency beam patterns, the pumped sound is located near $\theta = 4.5°$, and the scattered sound is located near $\theta = 7°$. Note that the shape of the difference frequency beam pattern does not vary with range. In the sum frequency beam patterns, the pumped sound is again located near $\theta = 4.5°$, but the scattered sound is now located near $\theta = 3.5°$. Whereas the scattered sum frequency sound dominates the pumped sum frequency sound at $Z = 100$, the relative levels are reversed beyond $Z = 400$.

In Fig. 2 we show sum and difference frequency beam patterns at $Z = 10$ as the absorption length is varied from $|L_\pm| = \infty$ (no absorption) to $|L_\pm| = z_2$ (strong absorption). The pumped and scattered waves are located near the same angles as in Fig. 1. Increasing absorption (decreasing $|L_\pm|$) attenuates the pumped difference frequency sound faster than the scattered difference frequency sound, whereas the reverse is true for the sum frequency sound.

We conclude that the relative amplitudes of the pumped and scattered waves depend not only on the nature and geometry of the sources (e.g., ka of each source,

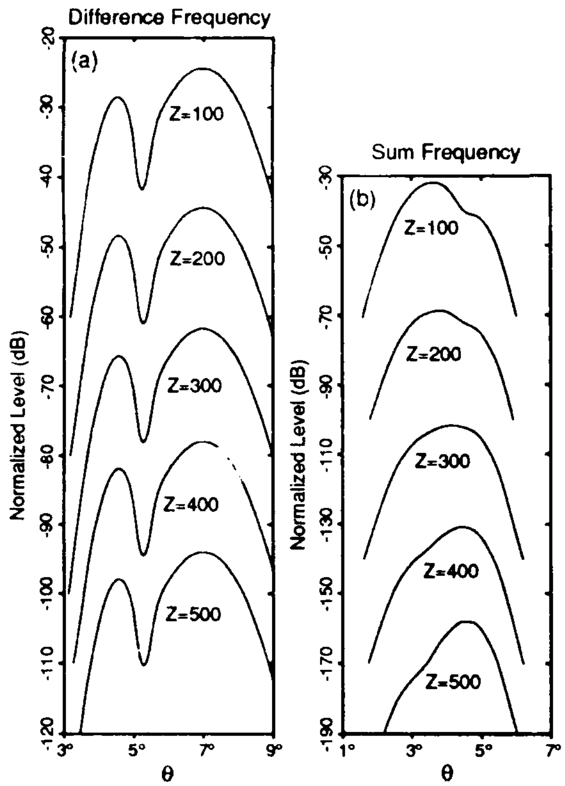


Figure 1

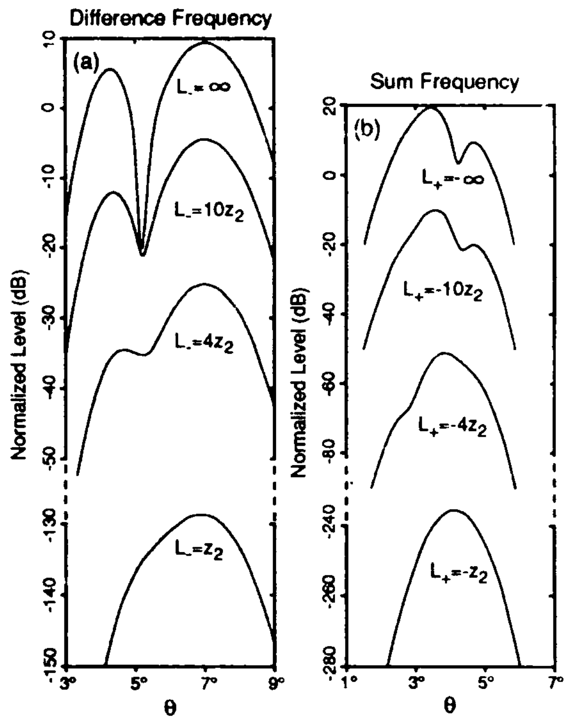


Figure 2

on-source pressure distributions, source separation, and beam interaction angle), but also on the dissipative properties of the medium through the sign of $L_{\pm}^{-1} = \alpha(\omega_1) + \alpha(\omega_2) - \alpha(\omega_1 \pm \omega_2)$. The function $\alpha(\omega)$ describes how the absorption coefficient of the medium varies with frequency. At distances of the order of the absorption length $|L_{\pm}|$ or larger, scattering of sound by sound is more likely to be observed at combination frequencies for which $L_{\pm} > 0$.

II. PARAMETRIC RECEIVING ARRAYS

The performance of a parametric receiving array near a reflecting surface has been investigated theoretically by Darvennes in parallel with project I. Naze Tjøtta (who is a member of Darvennes' doctoral committee) and Tjøtta also contributed to this work. Research into this problem is essentially complete. No new theoretical work has been performed since the publication of the third annual summary report[2] (July 1988) for ONR contract N00014-85-K-0708. Darvennes has used much of the last year to report her work in a dissertation that she is scheduled to defend in August 1989.

A. Background

The motivation is the potential use of a parametric receiving array for measuring freefield source directivities in reverberant environments. Particular attention is devoted to sound radiated from underwater sources near the air-water interface. Background for this work may be found in the second annual summary report[1] for ONR contract N00014-85-K-0708.

B. Results

We present here only one example that characterizes the results from this project. A comparison is made between two different methods for measuring a beam pattern with an omnidirectional hydrophone in the farfield of an underwater sound source. One is a conventional linear measurement, and the other is a parametric (nonlinear) measurement that makes use of intermodulation (difference frequency) components detected by the hydrophone. The comparison is made on the basis of which method better reproduces the beam pattern that would be measured in the absence of the reflecting surface, i.e., the freefield beam pattern.

As an example, we consider an axisymmetric source with radius 10 m that radiates in water at 250 Hz. The source is located at a depth of 50 m below the surface, and the sound is radiated in a direction that is parallel with the surface. The parametric array is formed by a pump with radius 1 m and frequency 50 kHz. The pump is located *on* the low frequency source, and an omnidirectional hydrophone located 2.6 km away on the axis of the pump measures not only the primary sound fields at 250 Hz and 50 kHz, but also the secondary (difference frequency) sound field at 49.75 kHz. To simplify the analysis it is assumed that the amplitude shading of both the source and the pump is Gaussian, and that the water is infinitely deep.

The results are shown in Fig. 3. The dotted line (with no ripples) is the freefield beam pattern of the 250 Hz source, and the broken line is the beam pattern at 250 Hz that would be measured linearly by the hydrophone in a plane that is perpendicular to the surface of the water. The ripples in the broken line result from the reflection of the 250 Hz sound from the surface of the water. The solid line is determined by the 49.75 kHz difference frequency sound that is measured by the hydrophone. Although the parametric array suppresses the ripples caused by the reflecting surface, it tends to overestimate the beamwidth of the low frequency source. Moreover, there is an optimal range for the separation between the pump and the hydrophone. As the separation is increased, the signal from the parametric array is increasingly affected by the multipath components, and as the separation is decreased, the beamwidth of the low frequency source is increasingly overestimated.

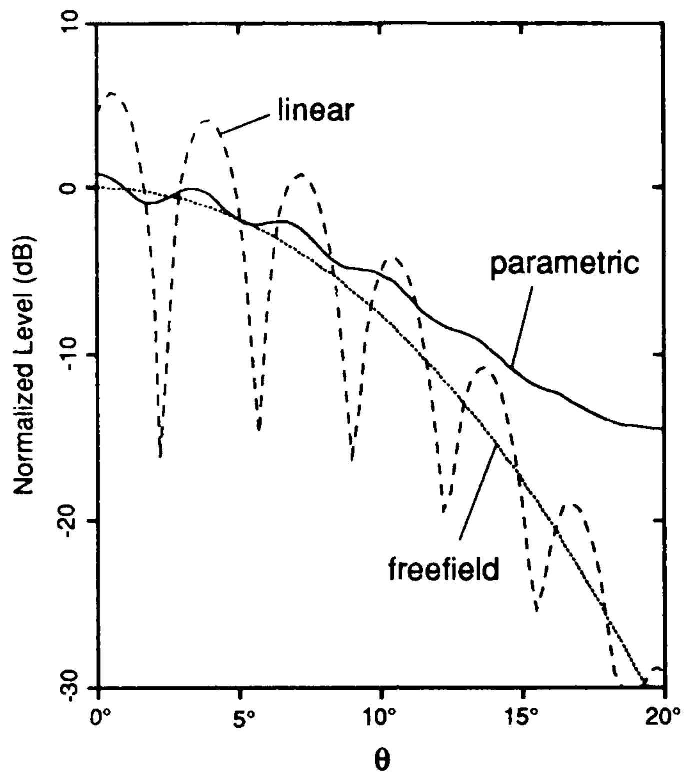


Figure 3

III. NONLINEAR EFFECTS IN ASYMMETRIC SOUND BEAMS

This is a new project that was begun in July 1988 by Kim. Kim received two months of support from the National Science Foundation (1 July 1988 through 31 August 1988) during the period covered by this report.

A. Background

The effects of asymmetry on nonlinear distortion are relevant to bottom and subbottom profiling with a parametric array, high intensity sound beams in shallow water, and caustic formation in inhomogeneous media. In general, asymmetries are introduced whenever a sound beam encounters an interface from a direction that deviates from the normal. For example, Muir et al.[7] have shown that near critical incidence, a parametric array attains greater bottom penetration through a water-sediment interface than does a comparable beam generated by a linear source. Theoretical explanations of this phenomenon have been offered by Jarzynski and Flax[8] and by Berktay et al.,[9] although diffraction effects are excluded in both analyses.[10,11,12] Only recently have appropriate theoretical models for diffract-

ing finite amplitude sound beams at interfaces and in inhomogeneous media been developed by the Tjøttas and their doctoral students (e.g., Ref. 13).

Our research is limited to two dimensional sound fields because of the relative ease with which numerical computations can be made at high intensities (in comparison with the same calculations for three dimensional nonaxisymmetric fields). Investigations based on two dimensional fields are nevertheless appropriate for waveguides, and also for comparison with recent analytical work on caustics by Marston.[14,15] McDonald and Kuperman[16] have recently produced numerical results for the water-borne propagation of a two dimensional finite amplitude pulse through a caustic.

B. Results

The work is based on a computer program for axisymmetric sound beams that was developed originally by Aanonsen et al.[17,18] References 19 and 20 discuss subsequent modifications of the program. The program has been modified by Kim for application to two dimensional asymmetric sound beams.

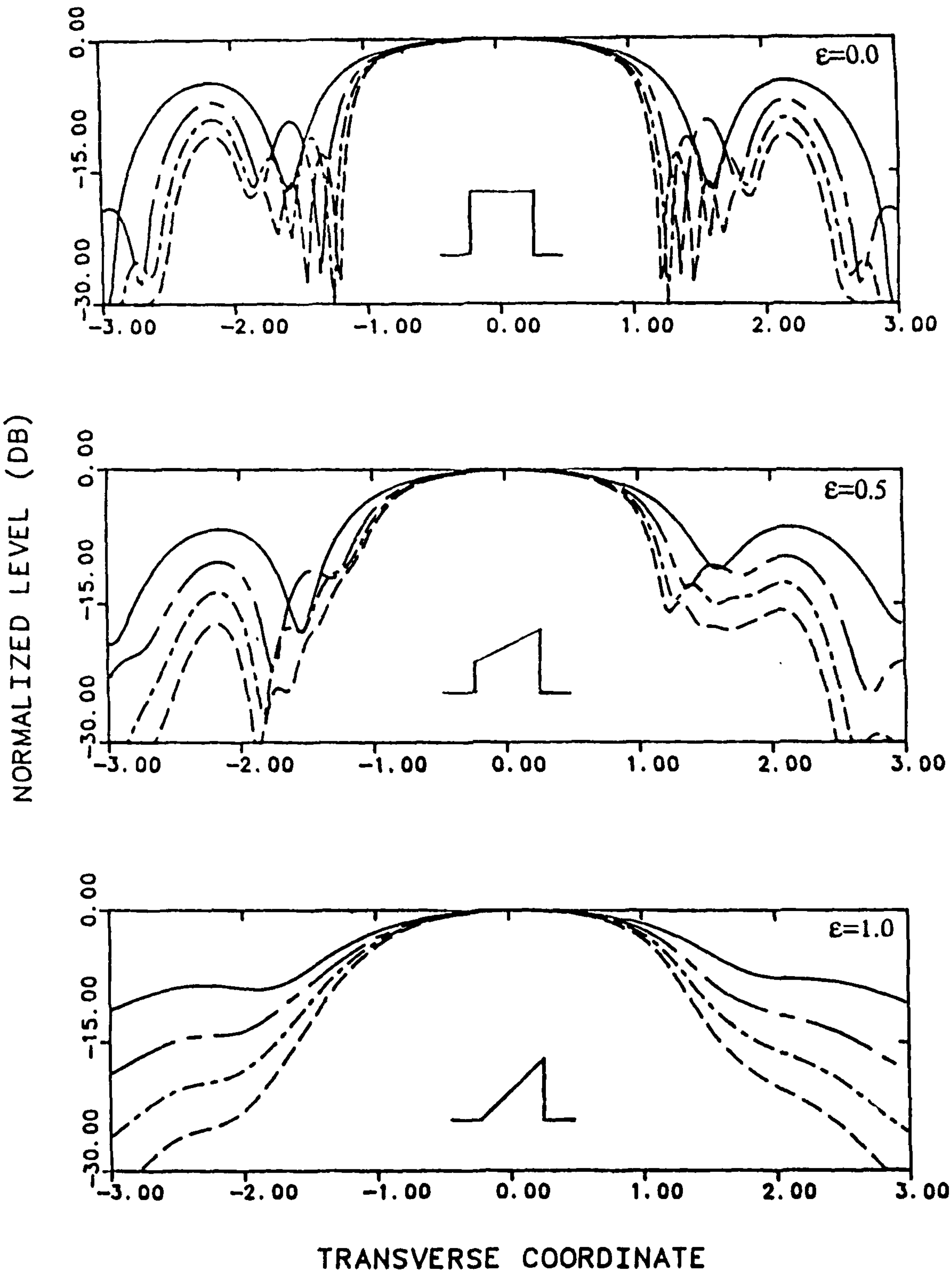
Shown in Fig. 4 are preliminary results for the beam patterns in a high intensity sound beam radiated by an infinite strip source. The solid lines are for the fundamental component, and the successively lower broken lines correspond to the nonlinearly generated higher harmonic components (second through fourth). The source excitation is time harmonic, and the pressure amplitude decreases linearly from p_0 at one edge to $(1-\varepsilon)p_0$ at the other, as shown schematically underneath each set of beam patterns. Source asymmetry is thus characterized by the dimensionless parameter ε. With $\varepsilon = 0$ the source has a uniform amplitude distribution, and the resulting beam patterns resemble those for the three dimensional case of a baffled piston. The effects of asymmetry at $\varepsilon = 0.5$ are most prominently manifested by the nonlinear nearfield phenomena known as fingers[21] (i.e., the additional sidelobes in the second, third, and fourth harmonic beam patterns). The asymmetry of the fingers increases with the number of the harmonic. At $\varepsilon = 1$, the fingers can no longer be observed, and the overall shapes of the beam patters are approximately symmetrical.

IV. PULSED FINITE AMPLITUDE SOUND BEAMS

This is a new project that was begun by Lee in September 1988. Lee received eight months of support from the Texas Advanced Research Program (1 September 1988 through 30 April 1989) during the period covered by this report.

A. Background

Although a broad theoretical base exists for the linear analysis of transient radiation in directive sound beams,[22,23] the investigation of transient effects in finite amplitude sound beams has progressed very slowly over the years. The first theoretical model of transient radiation from a directive finite amplitude source was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-30.00
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-2.00
-1.00
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2.00
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ε=0.0
ε=0.5
ε=1.0
TRANSVERSE COORDINATE

Figure 4

developed by Berktay[24] soon after Westervelt's discovery of the parametric array.[25] Berktay's model applies to a finite amplitude signal whose source waveform is a slowly modulated, high frequency carrier wave. Two simplifying assumptions are inherent in his analysis: (1) the primary beam behaves as a nondiffracting, collimated plane wave whose finite amplitude distortion is terminated within the nearfield by absorption, and (2) the results are valid only in the farfield. However, Berktay's prediction that the farfield time waveform should be described on axis by the second time derivative of the square of the source envelope function seemed to be verified by the classic experimental work of Moffett and coworkers.[26,27,28]

Subsequent extensions of Berktay's original analysis have been few and far between. Singhal and Zornig[29] considered the inverse problem, the calculation of the source waveform required to obtain a desired farfield response. Trivett and Rogers[30] investigated theoretically the second order sound that is generated by the nonlinear interaction of a pulsed beam which intersects the path of a monofrequency plane wave at an arbitrary angle. In a later paper[31] the authors extended their analysis to include the case where the monofrequency wave is a collimated plane wave beam. The collinear interaction of nondiffracting, collimated plane wave pulses was also investigated by Pace and Ceen[32] and, more recently, by Stepanishen and Koenigs.[33] Common to each of these analyses, however, are the same restrictions which apply to Berktay's work, namely, that the primary waves experience no diffraction, that the results apply only to the farfield radiation, and that the beam exhibits only moderately nonlinear effects.

The first theoretical analyses to take into account the diffraction of pulsed finite amplitude sound beams were performed in the Soviet Union. Zabolotskaya and Khokhlov,[34] in their paper where they derive the lossless nonlinear parabolic wave equation which bears their names, present a theoretical analysis of simple unipolar pulses in diffracting sound beams. Zhileikin and Rudenko,[35] using a numerical solution of the Khokhlov-Zabolotskaya equation,[36] investigated unipolar and bipolar Gaussian pulses in finite amplitude Gaussian beams. However, the more practical case of tone bursts radiated from piston-like sources was not considered.

Frøysa, Naze Tjøtta, and Tjøtta[37,38] recently developed a quasilinear theory that takes into account the combined effects of diffraction, absorption, and moderate nonlinearity in pulsed sound beams. Their work shows that the classic result by Berktay,[24] which seemed to be confirmed experimentally,[26,27,28] becomes increasingly inaccurate as the number of cycles in the pulse is reduced (i.e., for short tone bursts). The apparent experimental confirmation of Berktay's theory was due to the fact that relatively long tone bursts were used. Results from the quasilinear theory will provide useful comparisons with the results obtained from the algorithm being developed by Lee for strong finite amplitude pulses.

To date, there are no analytical solutions that accurately describe the combined effects of diffraction, absorption, and strong nonlinearity in sound beams. Instead, numerical solutions are used. The most common model equation for finite amplitude sound beams is the Khokhlov-Zabolotskaya-Kuznetsov (KZK) nonlinear parabolic equation. An algorithm developed by Aanonsen[17] that solves the KZK equation in the frequency domain has been used with great success for

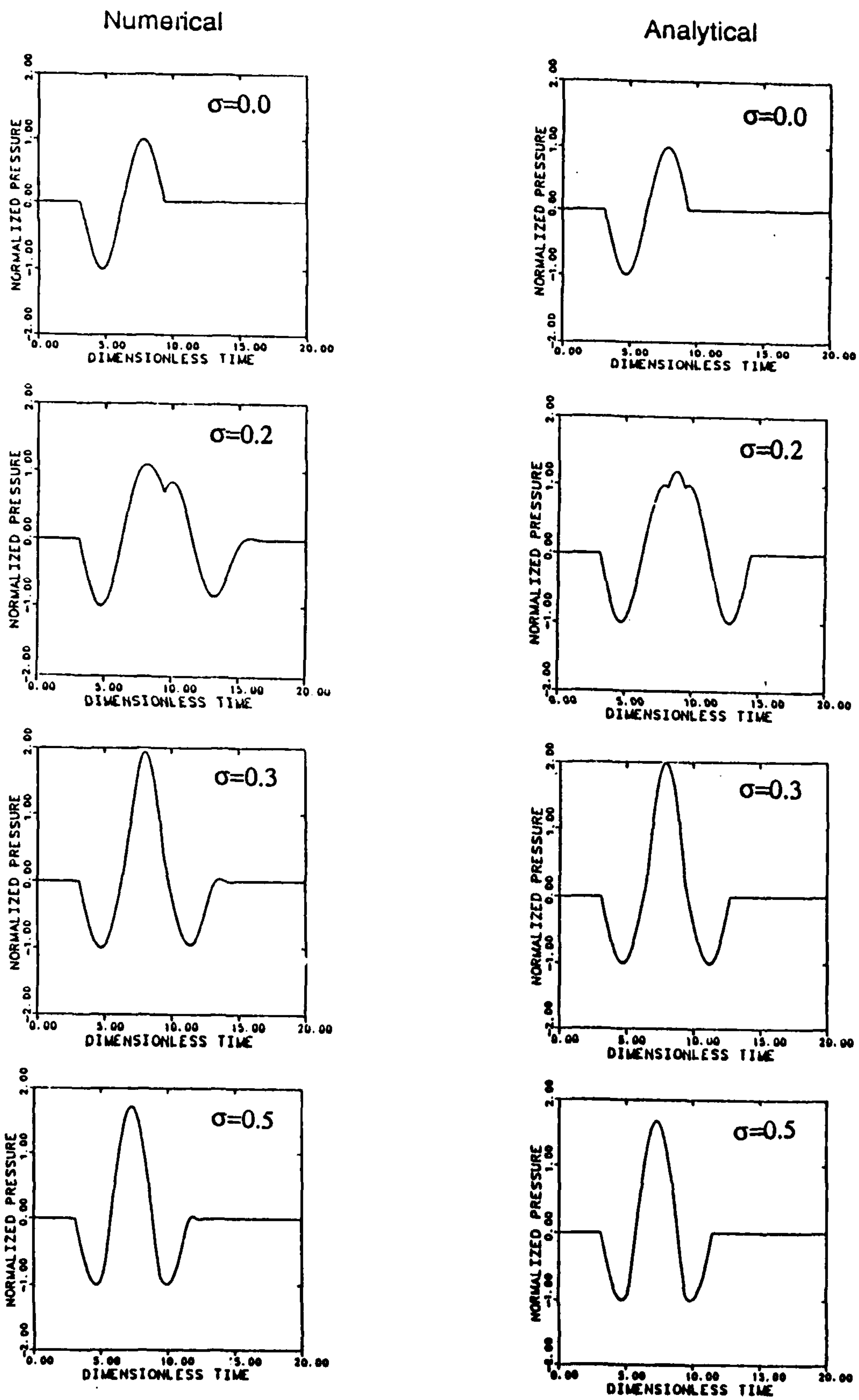
monofrequency[18,19,39,40] and bifrequency[41,42] sources. However, the number of harmonics that must be retained to describe the distortion in pulsed sound beams may require prohibitive amounts of computer time. Bjørnø and Neighbors[43] have recently developed a scheme by which the Aanonsen algorithm may be used for certain pulsed sound beams. To reduce the number of harmonics required to describe a single pulse, they consider an infinite series of pulses that are spaced sufficiently far apart in time that adjacent pulses do not interact with each other. Nevertheless, pulses with short rise times are inherently difficult to describe numerically in the frequency domain, particularly when nonlinear effects are relatively strong.

An alternative approach is provided by a time domain algorithm that was developed by McDonald and Kuperman.[16,44] Their program solves a nonlinear parabolic wave equation that is similar to a KZK equation without the absorption term. However, their method for integrating the diffraction term can become unstable in the highly oscillatory nearfield of a baffled piston.

B. Results

Lee has developed a numerical algorithm that solves the KZK equation in the time domain. Very near the source, where the solution can oscillate very rapidly, the diffraction term is integrated separately from the dissipation and nonlinearity terms. The spatial integration of the diffraction term is accomplished with an *implicit backward finite difference* method (as in the Aanonsen algorithm), and a trapezoidal rule is used to integrate over time. The nonlinearity and dissipation terms are integrated simultaneously with a predictor-corrector method, as has been done for the Burgers equation.[45] Away from the source, a transformed version of the KZK equation is used, and all terms are integrated simultaneously with an alternating direction implicit method.

Lee is still in the process of checking his numerical results against exact analytical solutions for various limiting cases. Results from numerical integration of the dissipation and nonlinearity terms (i.e., the Burgers equation) are in excellent agreement with the Hopf-Cole analytical solution. Shown in Fig. 5 are results for the diffraction term alone, which presented the most difficulty for numerical integration. The case considered is the on-axis field of a baffled piston that radiates a single cycle of a sine wave, as shown in the top two figures. The left column shows the numerical solution and the right column shows the analytical solution. The dimensionless coordinate σ measures range in terms of the Rayleigh distance for an infinite sine wave. Beyond $\sigma \approx 0.3$, the numerical solution is in excellent agreement with the analytical solution, apart from a small numerical disturbance that appears at the trailing edge of the pulse. Solutions for the highly oscillatory field in the neighborhood of baffled piston sources are also difficult to obtain when the KZK equation is integrated numerically in the frequency domain.[18]



Numerical
Analytical
σ=0.0
σ=0.2
σ=0.3
σ=0.5
NORMALIZED PRESSURE
DIMENSIONLESS TIME

Figure 5

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